Thermal conductivity for an ideal gas

For an ideal gas the following results are valid

\[ \overline{u} = \sqrt{\frac{8kT}{\pi m}} \]
\[ Z = \frac{1}{4} n\overline{u} \]
\[ \lambda = \frac{1}{2\sqrt{2\pi d^2 n}} \]

where \( \overline{u} \) [m s\(^{-1}\)] is mean velocity of the gas molecules, \( k \) [M L\(^2\) s\(^{-2}\) T] the Boltzmann constant, \( T \) the absolute temperature, \( m \) [M] the mass of one molecule of gas, \( n \) [L\(^{-3}\)] the number of molecules per unit volume, \( \bar{e} \) [L] mean free path, \( d \) [L] the diameter of a molecule.

Consider a constant temperature gradient along the y-axis. Molecules reaching a given plane have, on the average, had their last collision at a distance \( a \) [L] from the plane given by

\[ a = \frac{2}{3} \lambda \]

The mean translational energy of a molecule is given by

\[ \frac{1}{2} m \overline{u}^2 = \frac{3}{2} kT \]

and the specific heat per constant volume (per mol) \( C_V \) [M L\(^2\) s\(^{-2}\) T\(^{-1}\) mole\(^{-1}\)] by

\[ C_V = \left( \frac{\partial U}{\partial T} \right)_{V} = n \frac{d}{dT} \left( \frac{1}{2} m \overline{u}^2 \right) = \frac{3}{2} R \]

where \( U \) [L\(^2\) s\(^{-2}\)] is the internal energy per unit mass and \( R \) [M L\(^2\) s\(^{-2}\) T\(^{-1}\) M\(^{-1}\)] the gas constant.

The heat flow \( q_y \) [W L\(^{-2}\)] across a plane of constant \( y \) is found by summing the kinetic energies of the molecules that cross the plane per unit time in the positive \( y \)-direction and subtract the kinetic energy of the equal number of molecules that cross the plane in the negative \( y \)-direction.
\[ q_y = Z \left( \frac{1}{2} m u^2 \bigg|_{y-a} - \frac{1}{2} m u^2 \bigg|_{y+a} \right) \]

\[ = \frac{3}{2} kZ (T_{y-a} - T_{y+a}) \]

Expanding the temperature \( T \) in a Taylor’s series around \( y \) leads to

\[ T_{y-a} = T_y - \frac{2}{3} \lambda \frac{dT}{dy} \]

\[ T_{y+a} = T_y + \frac{2}{3} \lambda \frac{dT}{dy} \]

which by substituting in (5) leads to

\[ q_y = -\frac{1}{2} n k u \lambda \frac{dT}{dy} \]

which corresponds to Fourier’s law with for the conductivity \( k \) [W m\(^{-2}\) T\(^{-1}\)]

\[ k = \frac{1}{2} n k u \lambda = \frac{1}{3} C_v u \lambda = \frac{2}{3\pi} \frac{\sqrt{\pi} mkT}{\pi d^2} C_v \]