LECTURE 4

Computational Fluid Dynamics (CFD) wb1428

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- relevance of CFD
- first Fluent exercise
- advection and diffusion
- discretisation
- second Fluent exercise: advection of pollution
- discretisation of advection-diffusion
- first matlab exercise: ODE and PDE
- instability

$$\frac{dC}{dt} = -KC$$

$$\frac{\partial C}{\partial t} = -u\frac{\partial C}{\partial x} + K\frac{\partial^2 C}{\partial x^2}$$

- model equations for N-S
- solutions to model equations
- discretisation of model equations

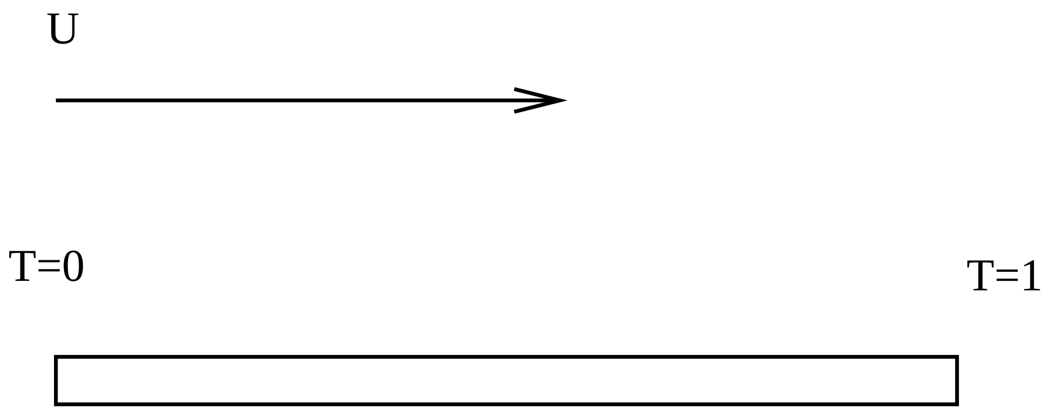
Navier-Stokes equations (N-S)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

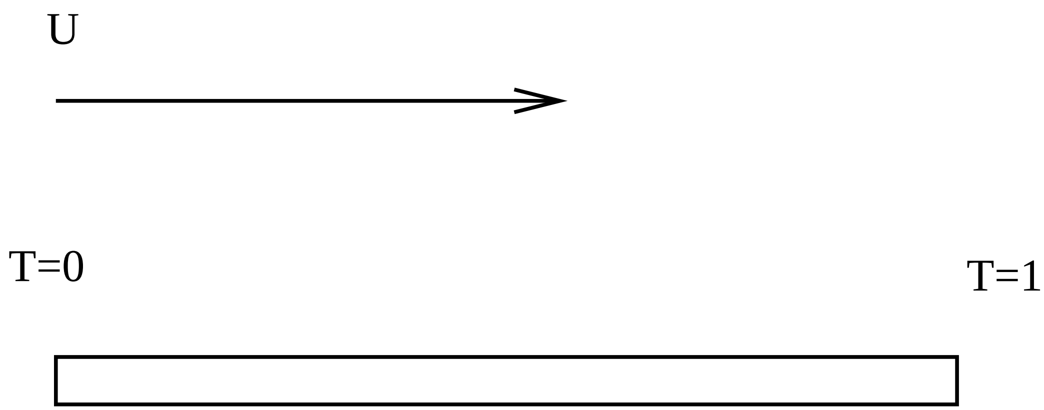
$$\begin{aligned}
\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= \frac{-1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial x^2} + \nu\frac{\partial^2 u}{\partial y^2} \\
I + ADVEC &= PRES + DIFF \\
\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} &= \frac{-1}{\rho}\frac{\partial p}{\partial y} + \nu\frac{\partial^2 v}{\partial x^2} + \nu\frac{\partial^2 v}{\partial y^2}
\end{aligned}$$

Some more facts about:

$$\frac{\partial C}{\partial t} = -u\frac{\partial C}{\partial x} + K\frac{\partial^2 C}{\partial x^2}$$

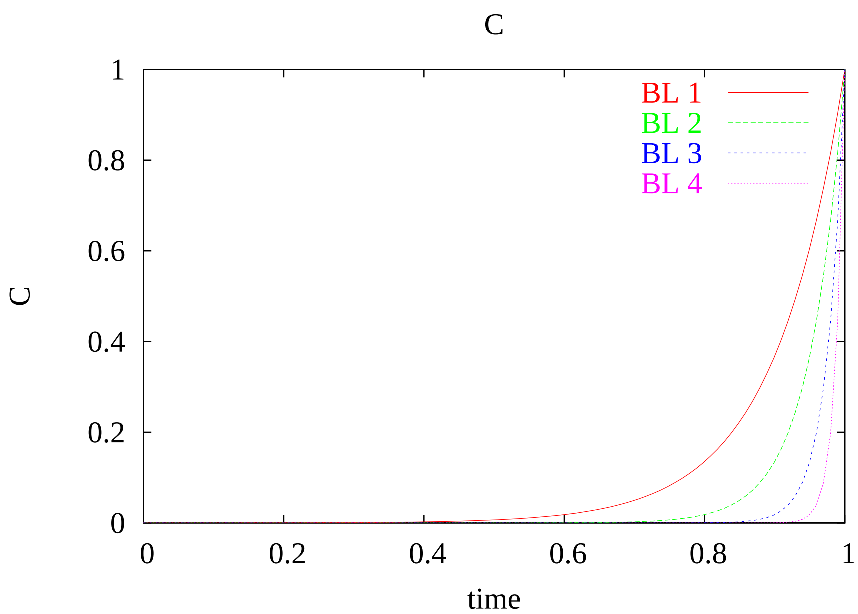


- BC
- only advection:
  - inflow only
- only diffusion or advection-diffusion:
  - inflow and outflow



- flow to right
- diffusion both directions
- relative importance advection diffusion
- $Pe = UL/K$

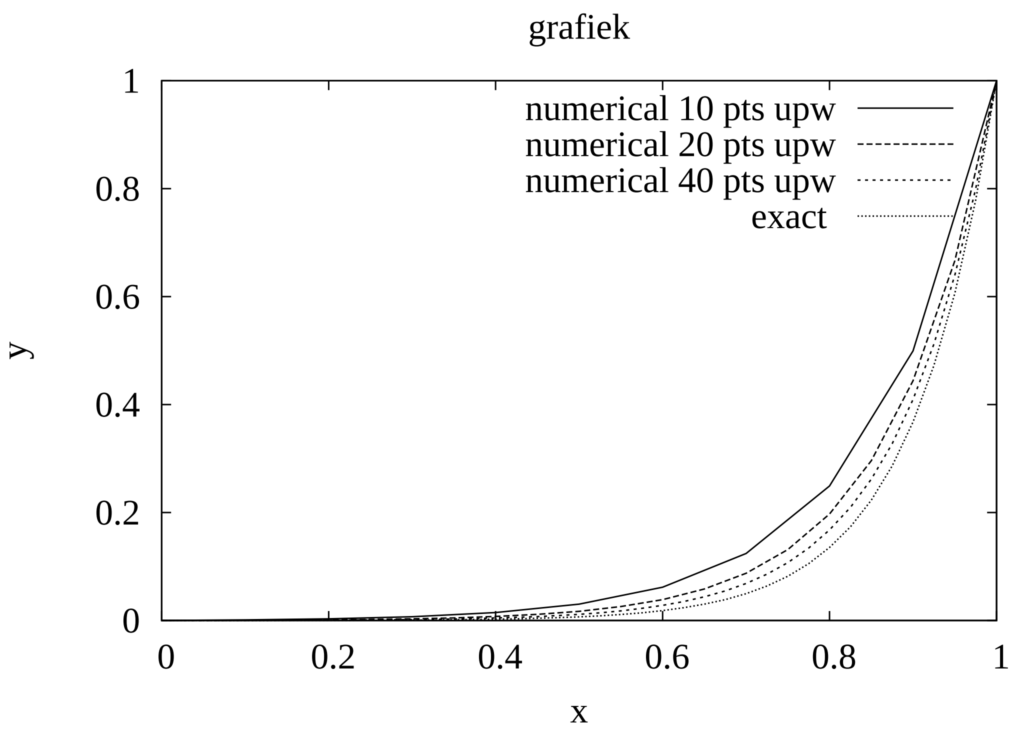
- Exact solution,stationary, $\frac{exp(x*u/K)-1}{exp(u/K)-1}$
- exercise: $u = 1$ and $k = 0.1$
- $Pe = 10$
- Boundary Layer $Pe = UL/K$



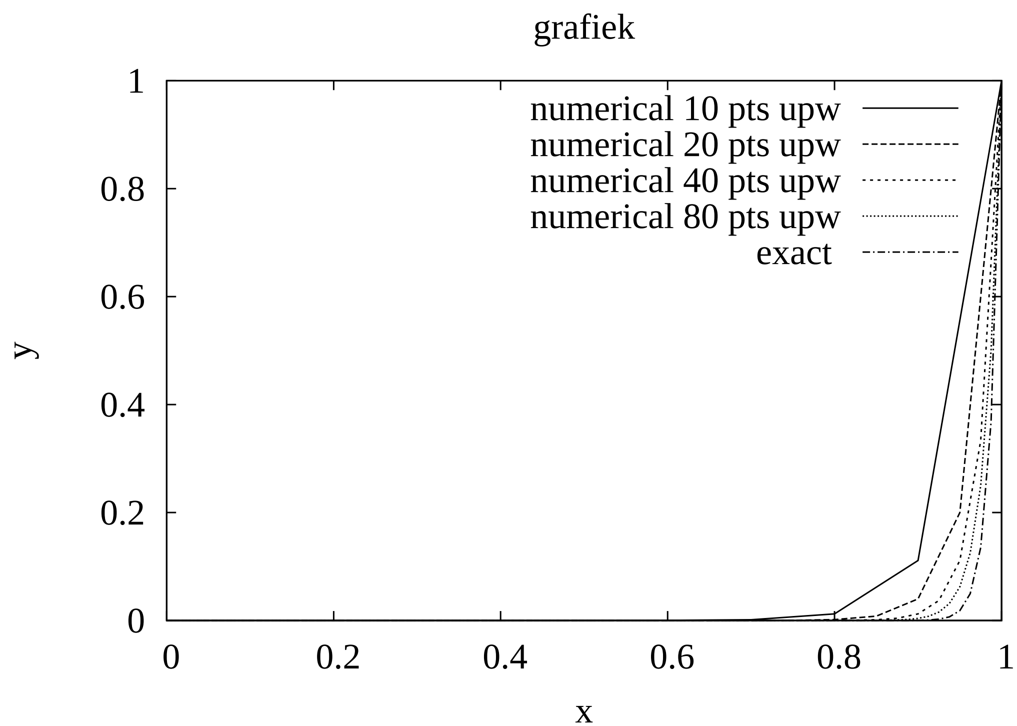
Exercise part 4, advection-diffusion

How well do we do numerically?

upwind advection, central diffusion Pe $= 10$



upwind advection, central diffusion Pe = 100



- numerical diffusion
- solution more accurate with more points
- upwind diffusion more important for high Pe

What about central differences?

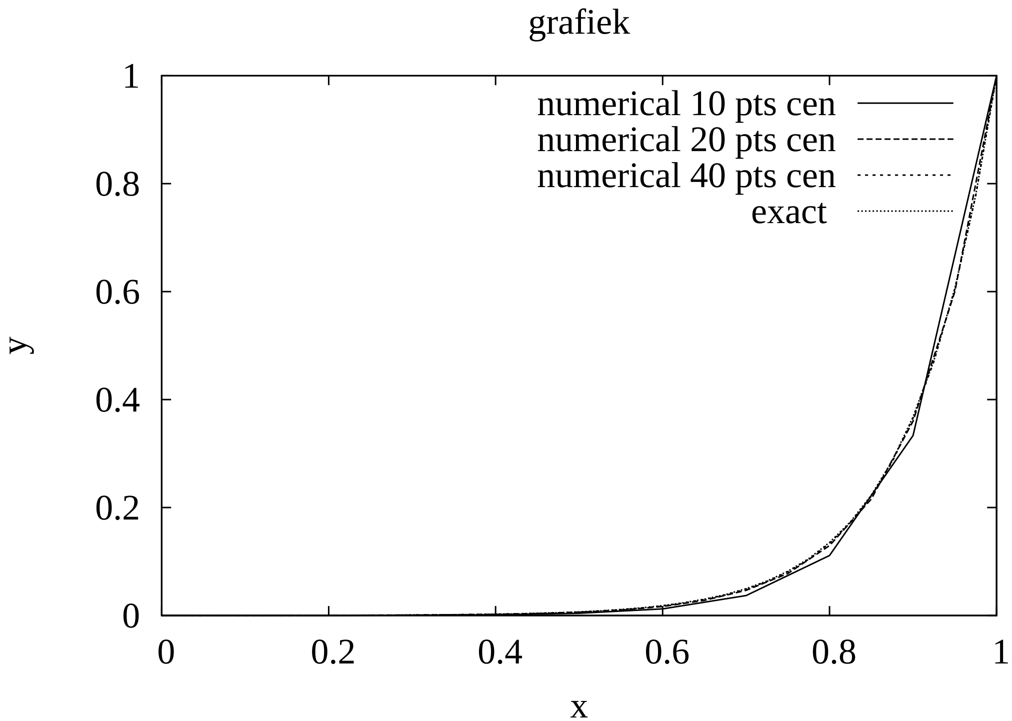
$$\frac{\partial C}{\partial x} \approx \frac{C(i+1) - C(i-1)}{2\Delta x}$$

NOTE a peculiarity with BC when $K = 0$!

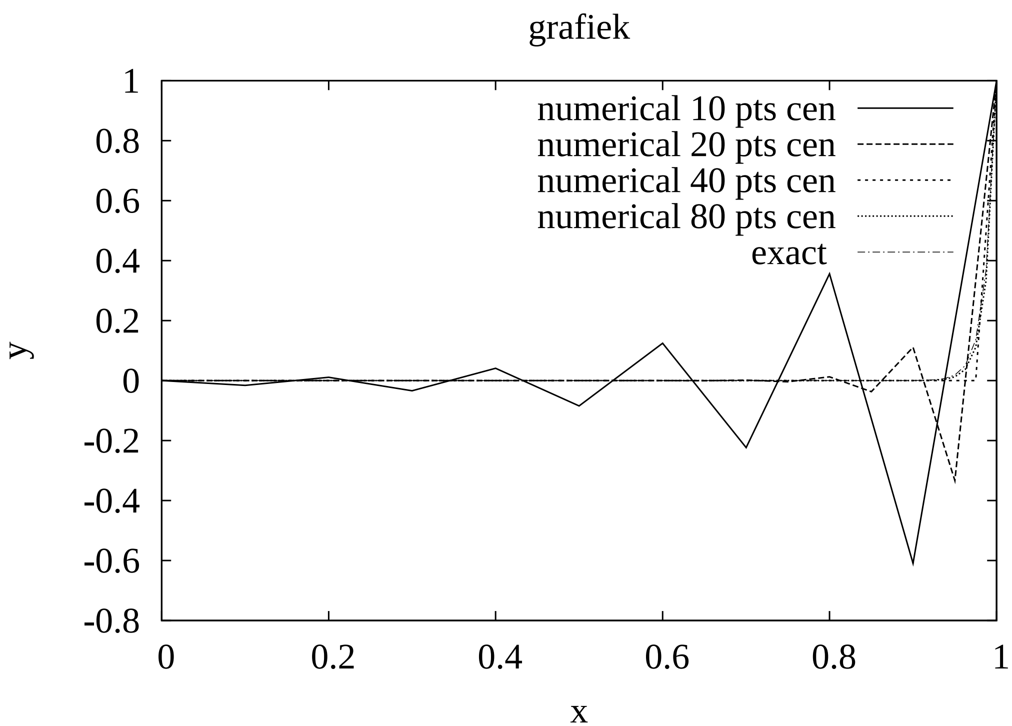
Only advection:

- advection equation: only BC at inflow
- central differences: also at RHS
- what will we take?

central advection, central diffusion Pe = 10



central advection, central diffusion Pe = 100



- numerical wiggles
- solution more accurate with more points
- dispersion more important for high Pe

Central differences wiggle up to certain grid cell size

- $cellPe = u\Delta x/K$
- $cellPe > 2$ CDS wiggles
- $cellPe < 2$ CDS NO wiggles

NOTE: these wiggles are not the same as instability!

Instability
ODE

$$Cnew = MCold$$

After last time you could solve

$$\frac{dC}{dt} = -KC$$

Numerical scheme:

$$\begin{aligned} \frac{C(t+\Delta t) - C(t)}{\Delta t} &= -KC(t) \\ C(t+\Delta t) &= C(t) - \Delta t K C(t) \\ C(t+\Delta t) &= (1 - \Delta t K) C(t) \end{aligned}$$

Program:

$$Cold = Cnew$$
$$Cnew = (1 - \Delta t K) Cold$$

or even:

$$C = (1 - \Delta t K) C$$

What about:

$$\frac{dC}{dt} = -KC + BC^2$$

$$\begin{aligned}
\frac{C(t+\Delta t) - C(t)}{\Delta t} &= -KC(t) + BC(t)^2 \\
C(t+\Delta t) &= C(t) - \Delta t KC(t) + BC(t)^2 \\
C(t+\Delta t) &= (1 - \Delta t K + BC(t))C(t)
\end{aligned}$$

$$\begin{aligned}
Cold &= Cnew \\
Cnew &= (1 - \Delta t K + BCold)Cold \\
C &= (1 - \Delta t K + BC)C
\end{aligned}$$

After today you can solve:

$$\frac{\partial C}{\partial t} = -u\frac{\partial C}{\partial x} + K\frac{\partial^2 C}{\partial x^2}$$

Then you can also do:

$$\frac{\partial C}{\partial t} = -C\frac{\partial C}{\partial x} + K\frac{\partial^2 C}{\partial x^2}$$

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} + K\frac{\partial^2 u}{\partial x^2}$$

You can almost do Navier-Stokes!

- 2D
- system
- coupled
- pressure